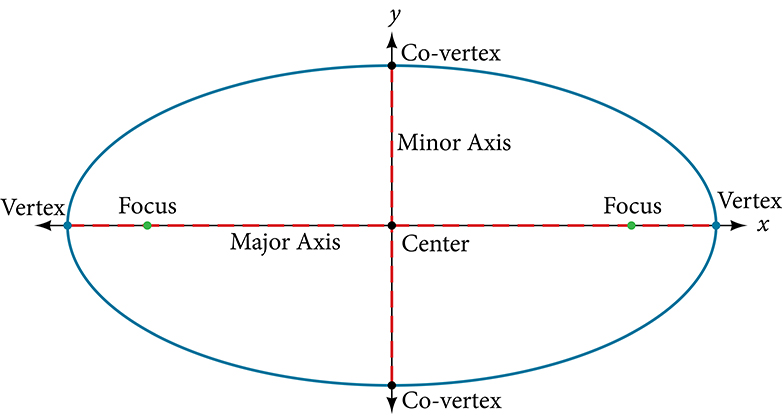
A conic section, or **conic**, is a shape resulting from intersecting a right circular cone with a plane. The angle at which the plane intersects the cone determines the shape, as shown below.

Planes intersecting right circular cones (which look like two ice cream cones stacked tip to tip). The intersection of a plane and a right circular cone is an ellipse, a hyperbola, or a parabola.
https://openstax.org/resources/124f5359a108b66bca16976a55f28da0e929ff9c

# Writing Equations of Ellipses in Standard Form

An **ellipse** is the set of all points in a plane such that the sum of their distances from two fixed points is a constant. Each fixed point is called a **focus** (plural: **foci**).

Every ellipse has two axes of symmetry. The longer axis is called the **major axis**, and the shorter axis is called the **minor axis**. Each endpoint of the major axis is the **vertex** of the ellipse (plural: **vertices**), and each endpoint of the minor axis is a **co-vertex** of the ellipse. The **center of the ellipse** is the midpoint of both the major and minor axes. The axes are perpendicular at the center. The foci always lie on the major axis and the sum of the distances from the foci to any point on the ellipse (the constant sum) is greater than the distance between the foci.



This section focuses on ellipses that are positioned vertically or horizontally in the coordinate plane.

## Writing Equations of Ellipses Centered at the Origin in Standard Form

Standard forms of equations tell us about key features of graphs. The key features of the ellipse are its center, vertices, co-vertices, foci, and lengths and positions of the major and minor axes.

**The standard form of the equation of an ellipse with center and major axis on the -axis** is

where

•

• The length of the major axis is

• The coordinates of the vertices are

• The length of the minor axis is

• The coordinates of the co-vertices are

• The coordinates of the foci are , where (see figure (a))

**The standard form of the equation of an ellipse with center and major axis on the -axis** is

where

•

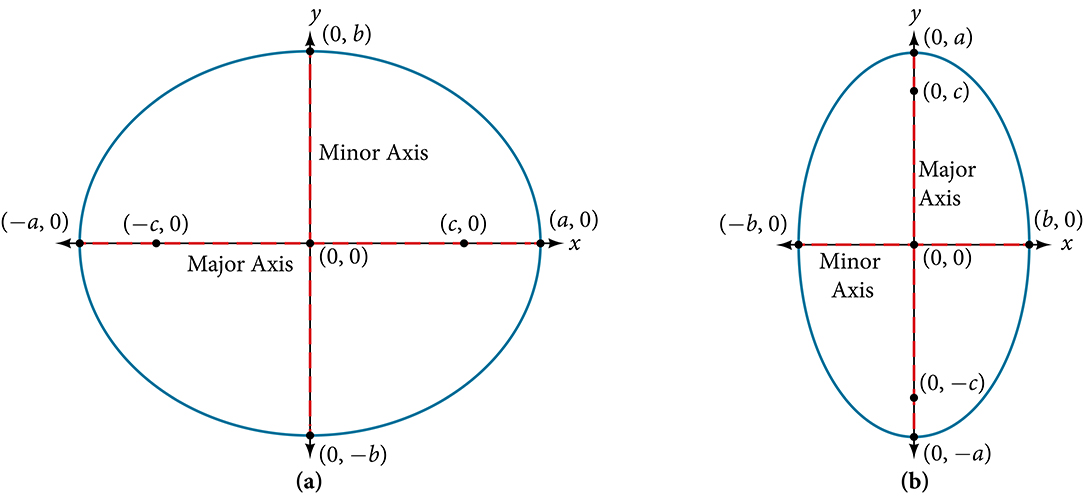
• The length of the major axis is

• The coordinates of the vertices are

• The length of the minor axis is

• The coordinates of the co-vertices are

• The coordinates of the foci are , where (see figure (b))



Note that the vertices, co-vertices, and foci are related by the equation . When we are given the coordinates of the foci and vertices of an ellipse, we can use this relationship to find the equation of the ellipse in standard form.

Given the vertices and foci of an ellipse centered at the origin, write its equation in standard form.

1) Determine whether the major axis lies on the or axis.

a. If the given coordinates of the vertices and foci have the form and respectively, then the major axis is the axis. Use the standard form .

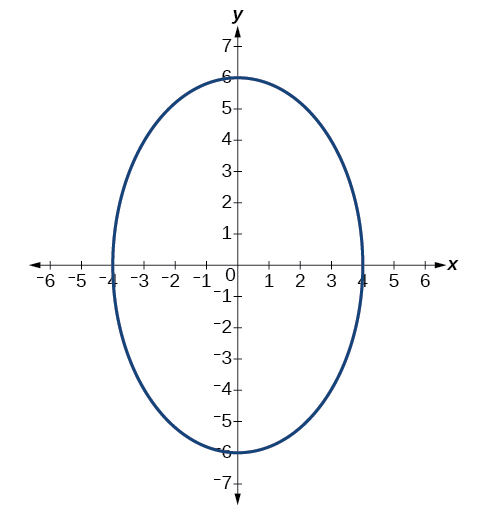
b. If the given coordinates of the vertices and foci have the form and respectively, then the major axis is the axis. Use the standard form .

2) Use the equation , along with the given coordinates of the vertices and foci, to solve for .

3) Substitute the values for and into the standard form of the equation determined in Step 1.

Examples

1. For the each of the following, determine whether the given equations represent ellipses. If yes, write in standard form.
2. What is the standard form equation of the ellipse that has vertices and foci ?
3. What is the standard form equation of the ellipse that has vertices and foci ?
4. Given the graph of the ellipse, determine its equation.



## Writing Equations of Ellipses Not Centered at the Origin

**The standard form of the equation of an ellipse with center and major axis on the -axis** is

where

•

• The length of the major axis is

• The coordinates of the vertices are

• The length of the minor axis is

• The coordinates of the co-vertices are

• The coordinates of the foci are , where (see figure (a))

**The standard form of the equation of an ellipse with center and major axis on the -axis** is

where

•

• The length of the major axis is

• The coordinates of the vertices are

• The length of the minor axis is

• The coordinates of the co-vertices are

• The coordinates of the foci are , where (see figure (b))

Ellipses centered at (h, k) with major and minor axes
https://openstax.org/resources/2580bcb23c74884f0b7ad07552b2d7b8a4fb4d0e

Just as with ellipses centered at the origin, ellipses that are centered at a point have vertices, co-vertices, and foci are related by the equation . We can use this relationship along with the midpoint and distance formulas to find the equation of the ellipse in standard form when the vertices and foci are given.

Given the vertices and foci of an ellipse not centered at the origin, write its equation in standard form.

1) Determine whether the major axis is parallel to the - or -axis.

a. If the -coordinates of the given vertices and foci are the same, then the major axis is parallel to the -axis. Use the standard form .

b. If the -coordinates of the given vertices and foci are the same, then the major axis is parallel to the-axis. Use the standard form .

2) Identify the center of the ellipse using the midpoint formula and the given coordinates for the vertices.

3) Find by solving for the length of the major axis, , which si the distance between the given vertices.

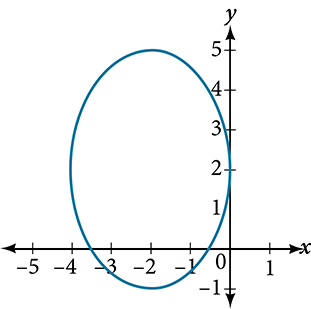
4) Find using and , found in Step 2, along with the given coordinates for the foci.

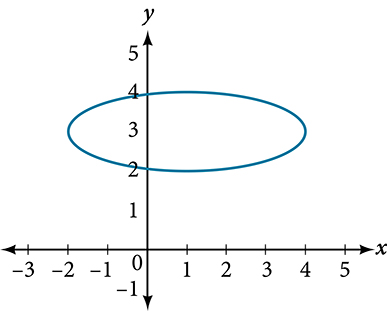
5) Solve for using the equation .

6) Substitute the values for and into the standard form of the equation determined in Step 1.

Examples

1. What is the standard form equation of the ellipse that has vertices and and foci and ?
2. What is the standard form equation of the ellipse that has vertices and and foci and ?
3. Given the graph of the ellipse, determine its equation.





# Graphing Ellipses

**Given the standard form of an equation for an ellipse centered at , sketch the graph.**

\*Note: if the center is at the origin then .

1) Use the standard forms of the equations of an ellipse to determine the center, position of the major axis, vertices, co-vertices, and foci.

a. If the equation is in the form , where , then

• The center is

• The major axis is parallel to the -axis

• The coordinates of the vertices are

• The coordinates of the co-vertices are

• The coordinates of the foci are

b. If the equation is in the form , where , then

• The center is

• The major axis is parallel to the -axis

• The coordinates of the vertices are

• The coordinates of the co-vertices are

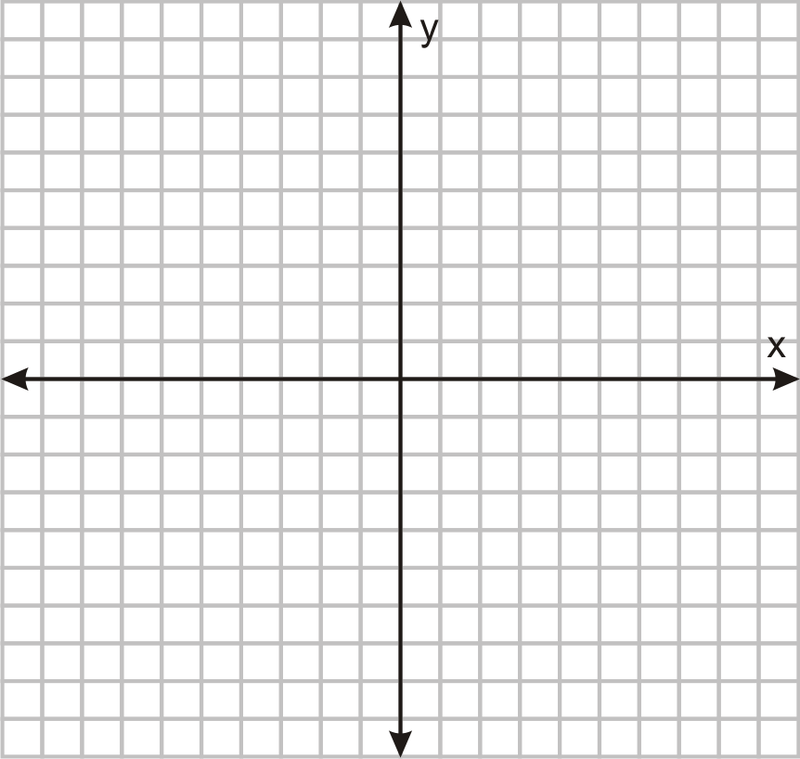
• The coordinates of the foci are

2) Solve for using the equation .

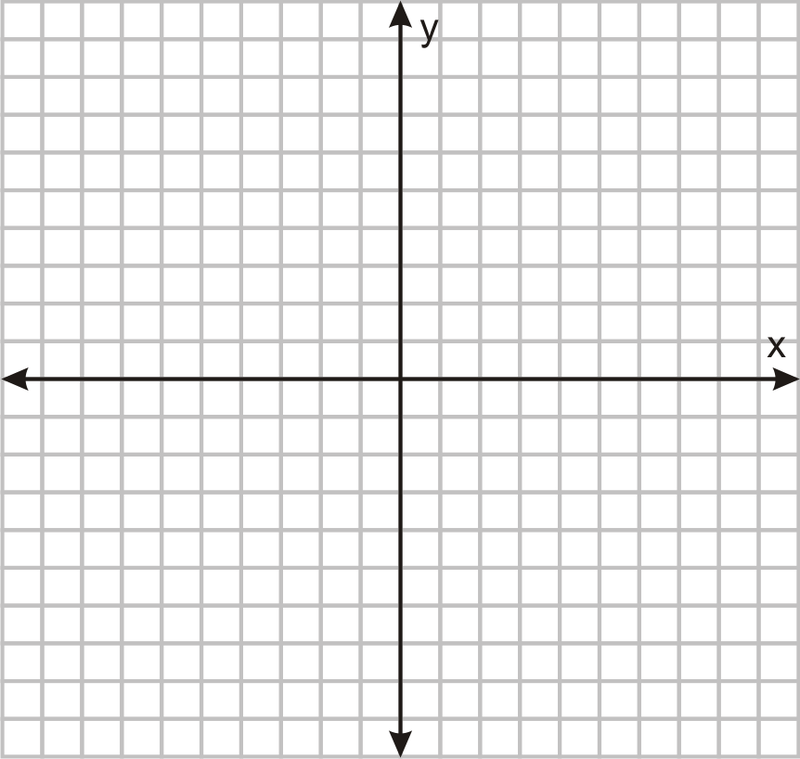
3) Plot the center, vertices, co-vertices, and foci in the coordinate plane, and draw a smooth curve to form the ellipse.

Examples

1. Graph the ellipse given by the equation, . Identify and label the center, vertices, co-vertices, and foci.



1. Graph the ellipse given by the equation . Rewrite the equation in standard form. Then identify and label the center, vertices, co-vertices, and foci.



1. Graph the ellipse given by the equation, . Identify and label the center, vertices, co-vertices, and foci.

